

Class XII Session 2024-25
Subject - Mathematics
Sample Question Paper - 4

Time Allowed: 3 hours

Maximum Marks: 80

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. Let $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then A^n is equal to [1]
 - a) $\begin{bmatrix} a^n & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$
 - b) $\begin{bmatrix} na & 0 & 0 \\ 0 & na & 0 \\ 0 & 0 & na \end{bmatrix}$
 - c) $\begin{bmatrix} a^n & 0 & 0 \\ 0 & a^n & 0 \\ 0 & 0 & a^n \end{bmatrix}$
 - d) $\begin{bmatrix} a^n & 0 & 0 \\ 0 & a^n & 0 \\ 0 & 0 & a \end{bmatrix}$
2. If the matrix $A = \begin{bmatrix} 3-2x & x+1 \\ 2 & 4 \end{bmatrix}$ is singular then $x = ?$. [1]
 - a) 1
 - b) 0
 - c) -1
 - d) -2
3. If A and B are invertible matrices, then which of the following is not correct? [1]
 - a) $(AB)^{-1} = B^{-1} A^{-1}$
 - b) $(A + B)^{-1} = B^{-1} + A^{-1}$
 - c) $\det(A)^{-1} = [\det(A)]^{-1}$
 - d) $\text{adj } A = |A| \cdot A^{-1}$
4. Let $f(x) = [x]^2 + \sqrt{x}$, where $[\bullet]$ and $\{\bullet\}$ respectively denotes the greatest integer and fractional part functions, then [1]
 - a) $f(x)$ is continuous and differentiable at $x = 0$
 - b) $f(x)$ is non differentiable $\forall x \in \mathbb{Z}$
 - c) $f(x)$ is discontinuous $\forall x \in \mathbb{Z} - \{1\}$
 - d) $f(x)$ is continuous at all integral points
5. Find the equation of the line which passes through the point (1, 2, 3) and is parallel to the vector $3\hat{i} + 2\hat{j} - 2\hat{k}$. [1]

$$\text{a) } \vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda (3\hat{i} + 2\hat{j} - 2\hat{k}),$$

$$\lambda \in R$$

$$\text{b) } \vec{r} = \widehat{2i} + 2\hat{j} + 3\hat{k} + \lambda (3\hat{i} + 2\hat{j} - 2\hat{k}).$$

$$\lambda \in R$$

$$\text{c) } \vec{r} = 4\hat{i} + 2\hat{j} + 3\hat{k} + \lambda (3\hat{i} + 2\hat{j} - 2\hat{k}).$$

$$\lambda \in R$$

$$\text{d) } \vec{r} = 3\hat{i} + 2\hat{j} + 3\hat{k} + \lambda (3\hat{i} + 2\hat{j} - 2\hat{k}).$$

$$\lambda \in R$$

6. The order of the differential equation of all circles of given radius a is: [1]

a) 4

b) 1

c) 2

d) 3

7. By graphical method solution of LLP maximize $Z = x + y$ subject to $x + y \leq 2x$; $y \geq 0$ obtained at [1]

a) at infinite number of points

b) only two points

c) only one point

d) at definite number of points

8. The domain of the function $\cos^{-1}(2x - 1)$ is [1]

a) $[0, \pi]$

b) $[-1, 1]$

c) $[0, 1]$

d) $(-1, 0)$

9. $\int_0^{\pi/2} \frac{\cos x}{(2+\sin x)(1+\sin x)} dx$ equals [1]

a) $\log\left(\frac{3}{4}\right)$

b) $\log\left(\frac{3}{2}\right)$

c) $\log\left(\frac{4}{3}\right)$

d) $\log\left(\frac{2}{3}\right)$

10. If $\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$, then the value of x is [1]

a) $\pm 6\sqrt{5}$

b) $5\sqrt{5}$

c) $\pm 4\sqrt{3}$

d) $\pm 3\sqrt{5}$

11. Objective function of an LPP is [1]

a) a function to be optimized

b) a function between the variables

c) a constraint

d) a relation between the variables

12. The vector in the direction of the vector $\hat{i} - 2\hat{j} + 2\hat{k}$ that has magnitude 9 is [1]

a) $\hat{i} - 2\hat{j} + 2\hat{k}$

b) $3(\hat{i} - 2\hat{j} + 2\hat{k})$

c) $9(\hat{i} - 2\hat{j} + 2\hat{k})$

d) $\frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}$

13. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ 3 & -1 & 9 \end{bmatrix}$, then the value of $\det(\text{Adj}(\text{Adj} A))$ equals [1]

a) 14641

b) 121

c) 11

d) 1331

14. If A and B are independent events such that $P(A) = \frac{1}{5}$, $P(A \cup B) = \frac{7}{10}$, then what is $P(\bar{B})$ equal to? [1]

a) $\frac{3}{8}$

b) $\frac{7}{9}$

c) $\frac{3}{7}$

d) $\frac{2}{7}$

15. Degree of the differential equation $\sin x + \cos\left(\frac{dy}{dx}\right) = y^2$ is [1]
 a) 2 b) not defined
 c) 0 d) 1
16. If $|\vec{a} \times \vec{b}| = 4$, $|\vec{a} \cdot \vec{b}| = 2$, then $|\vec{a}|^2 |\vec{b}|^2 =$ [1]
 a) 2 b) 20
 c) 8 d) 6
17. If $y = \tan^{-1} \frac{\cos x}{1 + \sin x}$ then $\frac{dy}{dx} = ?$ [1]
 a) $\frac{1}{2}$ b) 1
 c) 0 d) $\frac{-1}{2}$
18. The cartesian equation of a line is given by $\frac{2x-1}{\sqrt{3}} = \frac{y+2}{2} = \frac{z-3}{3}$ [1]
 The direction cosines of the line is
 a) $\frac{\sqrt{3}}{\sqrt{55}}, \frac{-4}{\sqrt{55}}, \frac{6}{\sqrt{55}}$ b) $\frac{3}{\sqrt{55}}, \frac{4}{\sqrt{55}}, \frac{6}{\sqrt{55}}$
 c) $\frac{\sqrt{3}}{\sqrt{55}}, \frac{4}{\sqrt{55}}, \frac{6}{\sqrt{55}}$ d) $\frac{-3}{\sqrt{55}}, \frac{4}{\sqrt{55}}, \frac{6}{\sqrt{55}}$
19. **Assertion (A):** If manufacturer can sell x items at a price of ₹ $(5 - \frac{x}{100})$ each. The cost price of x items is ₹ $(\frac{x}{5} + 500)$. Then, the number of items he should sell to earn maximum profit is 240 items. [1]
Reason (R): The profit for selling x items is given by $\frac{24}{5}x - \frac{x^2}{100} - 300$.
 a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.
 c) A is true but R is false. d) A is false but R is true.
20. **Assertion (A):** Let $A = \{1, 5, 8, 9\}$, $B = \{4, 6\}$ and $f = \{(1, 4), (5, 6), (8, 4), (9, 6)\}$, then f is a bijective function. [1]
Reason (R): Let $A = \{1, 5, 8, 9\}$, $B = \{4, 6\}$ and $f = \{(1, 4), (5, 6), (8, 4), (9, 6)\}$, then f is a surjective function.
 a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.
 c) A is true but R is false. d) A is false but R is true.

Section B

21. For the principal value, evaluate $\cot[\sin^{-1}\{\cos(\tan^{-1} 1)\}]$ [2]
 OR
 Which is greater, $\tan 1$ or $\tan^{-1} 1$?
22. Show that $f(x) = \sin x - \cos x$ is an increasing function on $(\frac{-\pi}{4}, \frac{\pi}{4})$. [2]
23. A stone is dropped into a quiet lake and waves move in circles at a speed of 4 cm/s. At the instant, when the radius of the circular wave is 10 cm, how fast is the enclosed area increasing? [2]
 OR
 Show that the function $f(x) = x^{100} + \sin x - 1$ is increasing on the interval $(\frac{\pi}{2}, \pi)$
24. Evaluate: $\int \tan^3 x \sec^3 x \, dx$ [2]
25. Prove that the determinant $\begin{vmatrix} x & \sin \theta & \cos \theta \\ -\sin \theta & -x & 1 \\ \cos \theta & 1 & x \end{vmatrix}$ is independent of θ . [2]



Section C

26. Evaluate the integral: $\int_0^\pi \frac{x}{a^2 \cos^2 x + b^2 \sin^2 x} dx$ [3]
27. In answering a question on a multiple choice questions test with four choices in each question, out of which only one is correct, a student either guesses or copies or knows the answer. The probability that he makes a guess is $\frac{1}{4}$ and the probability the he copies is also $\frac{1}{4}$. The probability that the answer is correct, given that he copied it is $\frac{3}{4}$. Find the probability that he knows the answer to the question, given that he correctly answered it. [3]
28. Evaluate the definite integral $\int_0^{\frac{\pi}{4}} \frac{\sin x \cos x}{\cos^4 x + \sin^4 x} dx$ [3]

OR

Evaluate the definite integral: $\int_1^2 e^{2x} \left(\frac{1}{x} - \frac{1}{2x^2} \right) dx$

29. Solve the following differential equation $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2 y^2$, given that $y = 1$, when $x = 0$. [3]

OR

Find the particular solution of the differential equation $(xe^{x/y} + y)dx = x dy$, given that $y(1) = 0$

30. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}| = 3, |\vec{b}| = 4, |\vec{c}| = 5$ and each one of them being $\perp$ to the sum of the other two, find $|\vec{a} + \vec{b} + \vec{c}|$ [3]

OR

If $\vec{a} = (\hat{i} - \hat{j}), \vec{b} = (3\hat{j} - \hat{k})$ and $\vec{c} = (7\hat{i} - \hat{k})$, find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and for which $\vec{c} \cdot \vec{d} = 1$.

31. Find $\frac{dy}{dx}$ of the function $(\cos x)^y = (\cos y)^x$. [3]

Section D

32. Find the area of the region $\{(x, y) : x^2 + y^2 \leq 4, x + y \geq 2\}$. [5]
33. Let $A = \{1, 2, 3, \dots, 9\}$ and R be the relation in $A \times A$ defined by $(a, b) R (c, d)$ if $a + d = b + c$ for $(a, b), (c, d)$ in $A \times A$. Prove that R is an equivalence relation and also obtain the equivalence class $[(2, 5)]$. [5]

OR

Let $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$. Consider the function $f: A \Rightarrow B$ defined by $f(x) = \left(\frac{x-2}{x-3} \right)$. Is f one-one and onto? Justify your answer.

34. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ then show that $A^2 - 5A + 7I = 0$ and hence find A^4 . [5]
35. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere. [5]

OR

A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

Section E

36. Read the following text carefully and answer the questions that follow: [4]
- A shopkeeper sells three types of flower seeds A_1, A_2, A_3 . They are sold in the form of a mixture, where the proportions of these seeds are $4 : 4 : 2$ respectively. The germination rates of the three types of seeds are 45%, 60% and 35% respectively.



Based on the above information:

- Calculate the probability that a randomly chosen seed will germinate. (1)
- Calculate the probability that the seed is of type A2, given that a randomly chosen seed germinates. (1)
- A die is throw and a card is selected at random from a deck of 52 playing cards. Then find the probability of getting an even number on the die and a spade card. (2)

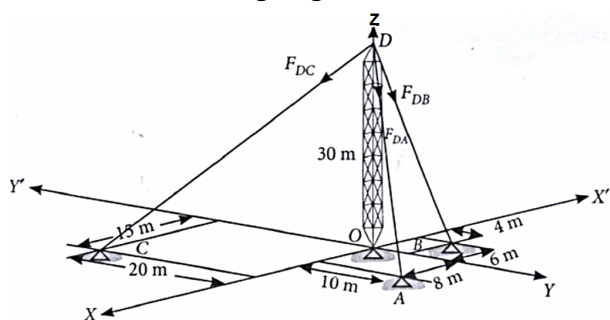
OR

If A and B are any two events such that $P(A) + P(B) - P(A \text{ and } B) = P(A)$, then find $P(A|B)$. (2)

37. **Read the following text carefully and answer the questions that follow:**

[4]

Consider the following diagram, where the forces in the cable are given.



- What is the equation of the line along cable AD? (1)
- What is length of cable DC? (1)
- Find vector DB (2)

OR

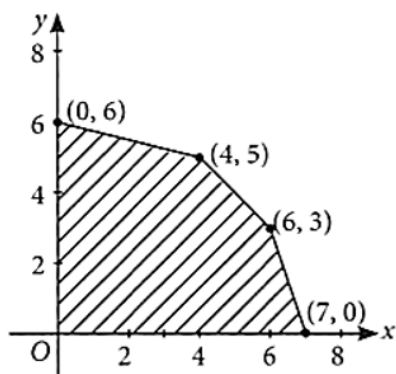
What is sum of vectors along the cable? (2)

38. **Read the following text carefully and answer the questions that follow:**

[4]

Linear programming is a method for finding the optimal values (maximum or minimum) of quantities subject to the constraints when a relationship is expressed as linear equations or inequations.

- At which points is the optimal value of the objective function attained? (1)
- What does the graph of the inequality $3x + 4y < 12$ look like? (1)
- Where does the maximum of the objective function $Z = 2x + 5y$ occur in relation to the feasible region shown in the figure for the given LPP? (2)



OR

What are the conditions on the positive values of p and q that ensure the maximum of the objective function $Z = px + qy$ occurs at both the corner points $(15, 15)$ and $(0, 20)$ of the feasible region determined by the given system of linear constraints? (2)



Solution

Section A

1.

$$(c) \begin{bmatrix} a^n & 0 & 0 \\ 0 & a^n & 0 \\ 0 & 0 & a^n \end{bmatrix}$$

$$\text{Explanation: } A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$$

$$A^n = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix} \times \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix} \times \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix} \times \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix} \dots \{n \text{ times, (where } n \in \mathbb{N})\}$$

$$A^n = \begin{bmatrix} a^n & 0 & 0 \\ 0 & a^n & 0 \\ 0 & 0 & a^n \end{bmatrix}$$

2. (a) 1

Explanation: When a given matrix is singular then the given matrix determinant is 0.

$$|A| = 0$$

$$\text{Given, } A = \begin{pmatrix} 3 - 2x & x + 1 \\ 2 & 4 \end{pmatrix}$$

$$|A| = 0$$

$$4(3 - 2x) - 2(x + 1) = 0$$

$$12 - 8x - 2x - 2 = 0$$

$$10 - 10x = 0$$

$$10(1 - x) = 0$$

$$x = 1$$

3.

$$(b) (A + B)^{-1} = B^{-1} + A^{-1}$$

Explanation: Since, A and B are invertible matrices.

So, we can say that

$$(AB)^{-1} = B^{-1} A^{-1} \dots (i)$$

$$\text{We know that, } A^{-1} = \frac{1}{|A|} (\text{adj } A)$$

$$\Rightarrow \text{adj } A = |A| \cdot A^{-1} \dots (ii)$$

$$\text{Also, } \det(A)^{-1} = [\det(A)]^{-1}$$

$$\Rightarrow \det(A)^{-1} = \frac{1}{[\det(A)]}$$

$$\Rightarrow \det(A) \cdot \det(A)^{-1} = 1 \dots (iii)$$

Which is true,

So, only option d is incorrect.

4.

$$(c) f(x) \text{ is discontinuous } \forall x \in \mathbb{Z} - \{1\}$$

Explanation: $f(x)$ is discontinuous $\forall x \in \mathbb{Z} - \{1\}$

$$5. (a) \vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(3\hat{i} + 2\hat{j} - 2\hat{k}), \lambda \in \mathbb{R}$$

Explanation: The equation of the line which passes through the point (1, 2, 3) and is parallel to the vector

$$3\hat{i} + 2\hat{j} - 2\hat{k}, \text{ let vector } \vec{a} = \hat{i} + \hat{j} + \hat{k} \text{ and vector } \vec{b} = 3\hat{i} + 2\hat{j} - 2\hat{k},$$

the equation of line is :

$$\vec{a} + \lambda \vec{b} = (\hat{i} + \hat{j} + \hat{k}) + \lambda(3\hat{i} + 2\hat{j} - 2\hat{k})$$

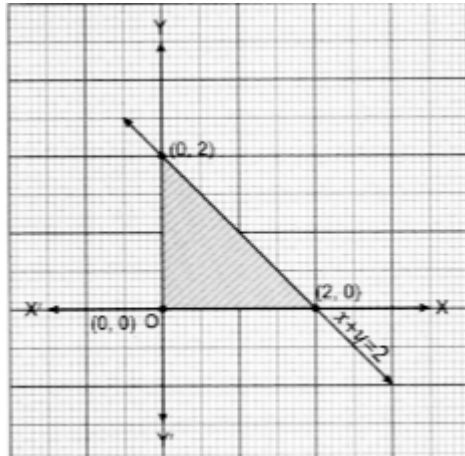
6.

(c) 2

Explanation: Let the equation of given family be $(x - h)^2 + (y - k)^2 = a^2$. It has two arbitrary constants h and k. Therefore, the order of the given differential equation will be 2.

7. (a) at infinite number of points

Explanation:



Feasible region is shaded region with corner points (0, 0), (2, 0) and (0, 2)

$$Z(0, 0) = 0$$

$$Z(2, 0) = 2 \leftarrow \text{maximise}$$

$$Z(0, 2) = 2 \leftarrow \text{maximise}$$

$Z_{\max} = 2$ obtained at (2, 0) and (0, 2) so is obtained at any point on line segment joining (2, 0) and (0, 2).

8.

(c) [0, 1]

Explanation: We have $f(x) = \cos^{-1}(2x - 1)$

$$\text{Since, } -1 \leq 2x - 1 \leq 1$$

$$\Rightarrow 0 \leq 2x \leq 2$$

$$\Rightarrow 0 \leq x \leq 1$$

$$\therefore x \in [0, 1]$$

9.

(c) $\log\left(\frac{4}{3}\right)$

Explanation: $\log\left(\frac{4}{3}\right)$

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\cos x}{(2 + \sin x)(1 + \sin x)} dx$$

$$\text{Let } \sin x = t, \text{ then } \cos x \, dx = dt$$

$$\text{When } x = 0, t = 0 \text{ } x = \frac{\pi}{2}, t = 1$$

Therefore the integral becomes

$$I = \int_0^1 \frac{dt}{(2+t)(1+t)}$$

$$= \int_0^1 \left[\frac{-1}{2+t} + \frac{1}{1+t} \right] dt$$

$$= [-\log(2+t) + \log(1+t)]_0^1$$

$$= [\log(1+t) - \log(2+t)]_0^1$$

$$= \log 2 - \log 3 - \log 1 + \log 2$$

$$= \log \frac{4}{3}$$

10.

(c) $\pm 4\sqrt{3}$

Explanation: Given, $\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$

$$\Rightarrow x \times 1 + (-5) \times 0 + (-1) \times 2 \times x \times 0 + (-5) \times 2 + (-1) \times 0$$

$$x \times 2 + (-5) \times 1 + (-1) \times 3 \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow [x - 2 \quad -10 \quad 2x - 8] \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow [(x - 2) \times x + (-10) \times 4 + (2x - 8) \times 1] = 0$$

$$\Rightarrow x^2 - 2x - 40 + 2x - 8 = 0$$

$$\Rightarrow x^2 = 48$$

$$\Rightarrow x = \pm\sqrt{48} = \pm 4\sqrt{3}$$

11. (a) a function to be optimized

Explanation: a function to be optimized

The objective function of a linear programming problem is either to be maximized or minimized i.e. objective function is to be optimized.

- 12.

(b) $3(\hat{i} - 2\hat{j} + 2\hat{k})$

Explanation: Let $\vec{a} = \hat{i} - 2\hat{j} + 2\hat{k}$

Unit vector in the direction of a vector $\vec{a}$

$$= \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{\sqrt{1^2 + 2^2 + 2^2}} = \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}$$

$\therefore$ Vector in the direction of $\vec{a}$ with magnitude 9

$$= 9 \cdot \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3} = 3(\hat{i} - 2\hat{j} + 2\hat{k})$$

13. (a) 14641

Explanation: We know that, for a square matrix of order n, if $|A| \neq 0$

$$\text{Adj}(\text{Adj } A) = |A|^{n-2} A \quad (\because n = 3)$$

$$\therefore \text{Adj}(\text{Adj } A) = |A|^{3-2} A \quad (\because n = 3)$$

$$= |A| A$$

$$\therefore |\text{Adj}(\text{Adj } A)| = ||A| A| = |A|^3 \det A = |A|^4$$

$$= 11^4 = 14641$$

14. (a) $\frac{3}{8}$

Explanation: Given that,

$$P(A) = \frac{1}{5}, P(A \cup B) = \frac{7}{10}$$

Also, A and B are independent events,

$$\therefore P(A \cap B) = P(A) \cdot P(B)$$

$$\Rightarrow P(A) + P(B) - P(A \cup B) = P(A) \cdot P(B)$$

$$\Rightarrow \frac{1}{5} + P(B) - \frac{7}{10} = \frac{1}{5} \times P(B)$$

$$\Rightarrow P(B) - \frac{P(B)}{5} = \frac{7}{10} - \frac{1}{5} = \frac{5}{10}$$

$$\Rightarrow \frac{4P(B)}{5} = \frac{1}{2} \Rightarrow P(B) = \frac{5}{8}$$

$$\therefore P(\bar{B}) = 1 - P(B) = 1 - \frac{5}{8} = \frac{3}{8}$$

- 15.

(b) not defined

Explanation: not defined

- 16.

(b) 20

Explanation: We know that

$$(\vec{a} \cdot \vec{b})^2 + (\vec{a} \times \vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2$$

$$|\vec{a}|^2 \cdot |\vec{b}|^2 = 2^2 + 4^2$$

$$|\vec{a}|^2 \cdot |\vec{b}|^2 = 20$$



17.

(d) $\frac{-1}{2}$

Explanation: Given that $y = \tan^{-1} \left(\frac{\cos x}{1 + \sin x} \right)$

Using $\cos x = \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}$, $\sin x = 2 \sin \frac{x}{2} \cos \frac{x}{2}$ and $\cos^2 \theta + \sin^2 \theta = 1$

Therefore,

$$y = \tan^{-1} \left(\frac{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}}{\cos^2 \frac{x}{2} + \sin^2 \frac{x}{2} + 2 \sin \frac{x}{2} \cos \frac{x}{2}} \right) = \tan^{-1} \left(\frac{(\cos \frac{x}{2} - \sin \frac{x}{2})(\cos \frac{x}{2} + \sin \frac{x}{2})}{(\cos \frac{x}{2} + \sin \frac{x}{2})^2} \right)$$

$$\Rightarrow y = \tan^{-1} \frac{\cos \frac{x}{2} - \sin \frac{x}{2}}{\cos \frac{x}{2} + \sin \frac{x}{2}}$$

Dividing by $\cos \frac{x}{2}$ in numerator and denominator, we get

$$y = \tan^{-1} \frac{1 - \tan \frac{x}{2}}{1 + \tan \frac{x}{2}}$$

Using $\tan \left(\frac{\pi}{4} - x \right) = \frac{1 - \tan x}{1 + \tan x}$, we obtain

$$y = \tan^{-1} \tan \left(\frac{\pi}{4} - \frac{x}{2} \right)$$

$$= \frac{\pi}{4} - \frac{x}{2}$$

Differentiating with respect to x, we

$$\frac{dy}{dx} = -\frac{1}{2}$$

18.

(c) $\frac{\sqrt{3}}{\sqrt{55}}, \frac{4}{\sqrt{55}}, \frac{6}{\sqrt{55}}$

Explanation: Rewrite the given line as

$$r \frac{2(x - \frac{1}{2})}{\sqrt{3}} = \frac{y+2}{2} = \frac{z-3}{3}$$

$$\text{or } \frac{x - \frac{1}{2}}{\sqrt{3}} = \frac{y+2}{4} = \frac{z-3}{6}$$

$\therefore$ DR's of line are $\sqrt{3}$, 4 and 6

Therefore, direction cosines are:

$$\frac{\sqrt{3}}{\sqrt{(\sqrt{3})^2 + 4^2 + 6^2}}, \frac{4}{\sqrt{(\sqrt{3})^2 + 4^2 + 6^2}}, \frac{6}{\sqrt{(\sqrt{3})^2 + 4^2 + 6^2}} \text{ or } \frac{\sqrt{3}}{\sqrt{55}}, \frac{4}{\sqrt{55}}, \frac{6}{\sqrt{55}}$$

19.

(c) A is true but R is false.

Explanation: Let $S(x)$ be the selling price of x items and let $C(x)$ be the cost price of x items.

Then, we have

$$S(x) = (5 - \frac{x}{100})x = 5x - \frac{x^2}{100}$$

$$\text{and } C(x) = \frac{x}{5} + 500$$

Thus, the profit function $P(x)$ is given by

$$P(x) = S(x) - C(x) = 5x - \frac{x^2}{100} - \frac{x}{5} - 500$$

$$\text{i.e. } P(x) = \frac{24}{5}x - \frac{x^2}{100} - 500$$

On differentiating both sides w.r.t. x, we get

$$P'(x) = \frac{24}{5} - \frac{x}{50}$$

Now, $P'(x) = 0$ gives $x = 240$.

$$\text{Also, } P'(x) = \frac{-1}{50}$$

$$\text{So, } P'(240) = \frac{-1}{50} < 0$$

Thus, $x = 240$ is a point of maxima.

Hence, the manufacturer can earn maximum profit, if he sells 240 items.

20.

(d) A is false but R is true.

Explanation: We have, $A = \{1, 5, 8, 9\}$, $B = \{4, 6\}$ and $f = \{(1, 4), (5, 6), (8, 4), (9, 6)\}$

So, all elements of B has a domain element on A or we can say elements 1 and 8 & 5 and 9 have some range 4 & 6, respectively.

Therefore, $f : A \rightarrow B$ is a surjective function not one to one function.

Also, for a bijective function, f must be both one to one onto.

Section B

21. We know that $\tan^{-1} 1 = \frac{\pi}{4}$.

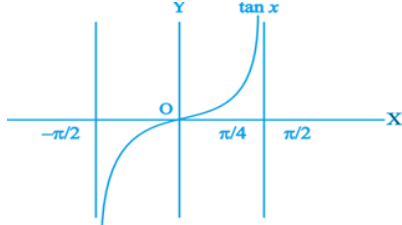
$$\begin{aligned} \therefore \cot[\sin^{-1}\{\cos(\tan^{-1} 1)\}] \\ = \cot\left\{\sin^{-1}\left(\cos\frac{\pi}{4}\right)\right\} = \cot\left(\sin^{-1}\frac{1}{\sqrt{2}}\right) = \cot\frac{\pi}{4} = 1 \end{aligned}$$

OR

From Fig. we note that $\tan x$ is an increasing function in the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$, since $1 > \frac{\pi}{4} \Rightarrow \tan 1 > \tan \frac{\pi}{4}$. This gives $\tan 1 > 1$

$$\Rightarrow \tan 1 > 1 > \frac{\pi}{4}$$

$$\Rightarrow \tan 1 > 1 > \tan^{-1}(1)$$



22. Given: $f(x) = \sin x - \cos x$

$$f'(x) = \cos x + \sin x$$

$$\begin{aligned} &= \sqrt{2} \left(\frac{1}{\sqrt{2}} \cos x + \frac{1}{\sqrt{2}} \sin x \right) \\ &= \sqrt{2} \left(\frac{\sin \pi}{4} \cos x + \frac{\cos \pi}{4} \sin x \right) \\ &= \sqrt{2} \sin \left(\frac{\pi}{4} + x \right) \end{aligned}$$

Now,

$$x \in \left(-\frac{\pi}{4}, \frac{\pi}{4} \right)$$

$$\Rightarrow -\frac{\pi}{4} < x < \frac{\pi}{4}$$

$$\Rightarrow 0 < \frac{\pi}{4} + x < \frac{\pi}{2}$$

$$\Rightarrow \sin 0^\circ < \sin \left(\frac{\pi}{4} + x \right) < \sin \frac{\pi}{2}$$

$$\Rightarrow 0 < \sin \left(\frac{\pi}{4} + x \right) < 1$$

$$\Rightarrow \sqrt{2} \sin \left(\frac{\pi}{4} + x \right) > 0$$

$$= f'(x) > 0$$

Hence, $f(x)$ is an increasing function on $(-\frac{\pi}{4}, \frac{\pi}{4})$

23. Let A be the area of the circle of radius r .

$$\text{Then, } A = \pi r^2$$

Therefore, the rate of change of area A with respect to time ' t ' is

$$\frac{dA}{dt} = \frac{d}{dt} (\pi r^2) = \frac{d}{dr} (\pi r^2) \cdot \frac{dr}{dt} = 2\pi r \frac{dr}{dt} \quad \dots (\text{By Chain Rule})$$

$$\text{Given that } \frac{dr}{dt} = 4 \text{ cm/s}$$

$$\text{Therefore, when } r = 10, \frac{dA}{dt} = 2\pi \times 10 \times 4 = 80\pi$$

Thus, the enclosed area is increasing at a rate of $80\pi \text{ cm}^2/\text{s}$, when $r = 10 \text{ cm}$.

OR

Given interval: $x \in (\pi/2, \pi)$

$$\Rightarrow \pi/2 < x < \pi$$

$$x^{99} > 1$$

$$100x^{99} > 100$$

$$\text{Again, } x \in (\pi/2, \pi) \Rightarrow -1 < \cos x < 0 \Rightarrow 0 > \cos x > -1$$

$$100x^{99} > 100 \text{ and } \cos x > -1$$

$$100x^{99} + \cos x > 100 - 1 = 99$$

$$100x^{99} + \cos x > 0$$

$$f'(x) > 0$$

Thus $f(x)$ is increasing on $(\pi/2, \pi)$

24. Let $I = \int \tan^3 x \sec^3 x \, dx$, then we have

$$I = \int \tan^2 x \sec^2 x (\sec x \tan x) \, dx = \int (\sec^2 x - 1) \sec^2 x (\sec x \tan x) \, dx$$

Substituting $\sec x = t$ and $\sec x \tan x \, dx = dt$, we obtain

$$I = \int (t^2 - 1) t^2 \, dt = \int (t^4 - t^2) \, dt = \frac{t^5}{5} - \frac{t^3}{3} + C = \frac{1}{5} \sec^5 x - \frac{1}{3} \sec^3 x + C$$

25. Let $\Delta = \begin{vmatrix} x & \sin \theta & \cos \theta \\ -\sin \theta & -x & 1 \\ \cos \theta & 1 & x \end{vmatrix}$

Expanding along first row,

$$\Delta = x \begin{vmatrix} -x & 1 \\ 1 & x \end{vmatrix} - \sin \theta \begin{vmatrix} -\sin \theta & 1 \\ \cos \theta & x \end{vmatrix} + \cos \theta \begin{vmatrix} -\sin \theta & -x \\ \cos \theta & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = x(-x^2 - 1) - \sin \theta(-x \sin \theta - \cos \theta) + \cos \theta(-\sin \theta + x \cos \theta)$$

$$\Rightarrow \Delta = -x^3 - x + x \sin^2 \theta + \sin \theta \cos \theta - \sin \theta \cos \theta + x \cos^2 \theta$$

$$\Rightarrow \Delta = -x^3 - x + x(\sin^2 \theta + \cos^2 \theta) = -x^3 - x + x = -x^3 \text{ which is independent of } \theta$$

Section C

26. We have,

$$I = \int_0^\pi \frac{x}{a^2 \cos^2 x + b^2 \sin^2 x} \, dx \dots (i)$$

Using property of definite integral we have,

$$= \int_0^\pi \frac{(\pi - x)}{a^2 \cos^2(\pi - x) + b^2 \sin^2(\pi - x)} \, dx$$

$$= \int_0^\pi \frac{\pi - x}{a^2 \cos^2 x + b^2 \sin^2 x} \, dx \dots (ii)$$

Adding (i) and (ii)

$$2I = \int_0^\pi \frac{x + \pi - x}{a^2 \cos^2 x + b^2 \sin^2 x} \, dx$$

$$= \pi \int_0^\pi \frac{1}{a^2 \cos^2 x + b^2 \sin^2 x} \, dx$$

$$= \pi \int_0^\pi \frac{\sec^2 x}{a^2 + b^2 \tan^2 x} \, dx \dots (\text{Dividing numerator and denominator by } \cos^2 x)$$

$$= 2\pi \int_0^{\frac{\pi}{2}} \frac{\sec^2 x}{a^2 + b^2 \tan^2 x} \, dx \dots \left[\text{Using } \int_0^{2a} f(x) \, dx = \int_0^a f(x) \, dx + \int_0^a f(2a - x) \, dx \right]$$

Put $\tan x = t$

$$\Rightarrow \sec^2 x \, dx = dt$$

When $x \rightarrow 0$; $t \rightarrow 0$

and $x \rightarrow \frac{\pi}{2}$; $t \rightarrow \infty$

$$\therefore 2I = 2\pi \int_0^{\frac{\pi}{2}} \frac{dt}{a^2 + b^2 t^2}$$

$$\Rightarrow I = \frac{\pi}{b^2} \int_0^{\frac{\pi}{2}} \frac{dt}{\frac{a^2}{b^2} + t^2}$$

$$= \frac{\pi}{b^2} \times \frac{b}{a} \left[\tan^{-1} \left(\frac{bt}{a} \right) \right]_0^\infty$$

$$= \frac{\pi}{ab} \left[\frac{\pi}{2} - 0 \right]$$

$$= \frac{\pi}{ab} \times \frac{\pi}{2}$$

$$= \frac{\pi^2}{2ab}$$

$$\text{Hence, } I = \frac{\pi^2}{2ab}$$

27. Let E_1 = Student guesses the answer

E_2 = Student copies the answer

E_3 = Student knows the answer

A = Student answers the question correctly.

$$P(E_1) = \frac{1}{4}, P(E_2) = \frac{1}{4}, P(E_3) = 1 - \left(\frac{1}{4} + \frac{1}{4} \right) = \frac{1}{2}$$

$$P(A | E_1) = \frac{1}{4}, P(A | E_2) = \frac{3}{4}, P(A | E_3) = 1$$

The required probability

$$= P(E_3 | A) = \frac{P(E_3) \times P(A | E_3)}{\sum_{i=1}^3 P(E_i) \times P(A | E_i)}$$

$$= \frac{\frac{1}{2} \times 1}{\frac{1}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{3}{4} + \frac{1}{2} \times 1}$$

$$= \frac{\frac{1}{8}}{\frac{1}{8} + \frac{3}{8} + 1} = \frac{\frac{1}{8}}{\frac{8}{8} + \frac{3}{8} + \frac{8}{8}} = \frac{\frac{1}{8}}{\frac{19}{8}} = \frac{1}{19}$$

$$28. I = \int_0^{\pi/4} \frac{\sin x \cdot \cos x}{\cos^4 x + \sin^4 x} dx$$

Dividing Nr. and Dr. by $\cos^4 x$

$$= \int_0^{\pi/4} \frac{\frac{\sin x \cdot \cos x}{\cos^4 x}}{\frac{\cos^4 x}{\cos^4 x} + \frac{\sin^4 x}{\cos^4 x}} dx$$

$$= \int_0^{\pi/4} \frac{\tan x \cdot \sec^2 x}{1 + \tan^4 x} dx$$

$$= \int_0^{\pi/4} \frac{\tan x \cdot \sec^2 x}{1 + (\tan^2 x)^2} dx$$

Put $\tan^2 x = t$

$$2 \tan x \cdot \sec^2 x dx = dt$$

When $x = 0$, $t = 0$ and when $x = \frac{\pi}{4}$, $t = 1$

$$\therefore I = \frac{1}{2} \int_0^1 \frac{dt}{1+t^2}$$

$$= \frac{1}{2} [\tan^{-1} t]_0^1$$

$$= \frac{1}{2} \cdot \frac{\pi}{4} = \frac{\pi}{8}$$

OR

We have,

$$I = \int_1^2 e^{2x} \left(\frac{1}{x} - \frac{1}{2x^2} \right) dx$$

$$I = \int_1^2 \frac{1}{x} \cdot e^{2x} - \int_1^2 \frac{1}{2x^2} \cdot e^{2x} dx$$

$$\Rightarrow I = I_1 - I_2$$

$$\text{Now, } I_1 = \int_1^2 \frac{1}{x} e^{2x} \text{ (By parts we have)}$$

$$\Rightarrow I_1 = \left[\frac{1}{x} \right]_1^2 \cdot \int_1^2 e^{2x} dx - \int_1^2 -\frac{1}{x^2} \cdot \frac{e^{2x}}{2} dx$$

$$\Rightarrow I_1 = \left[\frac{1}{x} \cdot \frac{e^{2x}}{2} \right]_1^2 + \int_1^2 \frac{1}{2x^2} e^{2x} dx$$

$$\Rightarrow I_1 = \left[\frac{1}{2x} e^{2x} \right]_1^2 + I_2$$

$$\text{As, } I = I_1 - I_2$$

$$\Rightarrow I = \left[\frac{1}{2x} e^{2x} \right]_1^2 - I_2 + I_2$$

$$\Rightarrow I = \left[\frac{1}{2x} e^{2x} \right]_1^2 = \frac{1}{2} \left[\frac{1}{2} e^4 - e^2 \right]$$

$$\Rightarrow I = \frac{1}{4} e^2 (e^2 - 1)$$

29. According to the question ,

Given differential equation is ,

$$\frac{dy}{dx} = 1 + x^2 + y^2 + x^2 y^2$$

$$\Rightarrow \frac{dy}{dx} = 1(1 + x^2) + y^2(1 + x^2)$$

$$\Rightarrow \frac{dy}{dx} = (1 + x^2)(1 + y^2)$$

$$\Rightarrow \frac{dy}{1+y^2} = (1 + x^2) dx$$

On integrating both sides, we get

$$\int \frac{dy}{1+y^2} = \int (1 + x^2) dx$$

$$\Rightarrow \tan^{-1} y = x + \frac{x^3}{3} + C \dots (i)$$

Given that $y = 1$, when $x = 0$.

On putting $x = 0$ and $y = 1$ in Eq. (i), we get

$$\tan^{-1} 1 = C$$

$$\Rightarrow \tan^{-1}(\tan \pi/4) = C \quad \left[\because \tan \frac{\pi}{4} = 1 \right]$$

$$\Rightarrow C = \frac{\pi}{4}$$

On putting the value of C in Eq. (i), we get

$$\tan^{-1} y = x + \frac{x^3}{3} + \frac{\pi}{4}$$

$$\therefore y = \tan\left(x + \frac{x^3}{3} + \frac{\pi}{4}\right)$$

which is the required solution of differential equation.

OR

The given differential equation can be rewritten as,

$$xe^{\frac{y}{x}} - y + x \frac{dy}{dx} = 0$$

$$\Rightarrow x \frac{dy}{dx} = y - xe^{\frac{y}{x}}$$

$$\Rightarrow \frac{dy}{dx} = \left(\frac{y}{x}\right) - e^{\frac{y}{x}}$$

$$\Rightarrow \frac{dy}{dx} = f\left(\frac{y}{x}\right)$$

$\Rightarrow$ the given differential equation is a homogenous equation.

The solution of the given differential equation is:

Put $y = vx$

$$\Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$v + x \frac{dv}{dx} = \left(\frac{vx}{x}\right) - e^{\frac{vx}{x}}$$

$$\Rightarrow x \frac{dv}{dx} = -e^v$$

$$\Rightarrow \frac{dv}{e^v} = \frac{-dx}{x}$$

Integrating both the sides we get:

$$\Rightarrow \int \frac{dv}{e^v} = - \int \frac{dx}{x} + c$$

$$\Rightarrow -e^{-v} = -\ln|x| + c$$

Resubstituting the value of $y = vx$ we get

$$\Rightarrow -e^{-\left(\frac{y}{x}\right)} = -\ln|x| + c$$

Now, $y(1) = 0$

$$\Rightarrow -e^{-(0)} = -\ln|1| + c$$

$$\Rightarrow c = -1$$

$$\Rightarrow \log|x| + e^{-y/x} = 1$$

$$30. \vec{a} \cdot (\vec{b} + \vec{c}) = 0, \vec{b} \cdot (\vec{c} + \vec{a}) = 0, \vec{c} \cdot (\vec{a} + \vec{b}) = 0 \text{ (Given)}$$

$$|\vec{a} + \vec{b} + \vec{c}|^2 = (\vec{a} + \vec{b} + \vec{c}) \cdot (\vec{a} + \vec{b} + \vec{c})$$

$$= \vec{a} \cdot \vec{a} + \vec{a} \cdot (\vec{b} + \vec{c}) + \vec{b} \cdot \vec{b} + \vec{b} \cdot (\vec{a} + \vec{c}) + \vec{c} \cdot \vec{c} + \vec{c} \cdot (\vec{a} + \vec{b})$$

$$= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2$$

$$= 9 + 16 + 25$$

$$= 50$$

$$|\vec{a} + \vec{b} + \vec{c}| = \sqrt{50}$$

$$= 5\sqrt{2}$$

OR

$$\text{Let } \vec{a} = a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}$$

$$\vec{d} \perp \vec{a}, \vec{d} \cdot \vec{a} = 0 \Rightarrow (a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}) (\hat{i} - \hat{j}) = 0$$

$$\Rightarrow a_1 - a_2 = 0 \dots(i)$$

$$\vec{d} \perp \vec{b}, \vec{d} \cdot \vec{b} = 0 \Rightarrow (a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}) (3\hat{j} - \hat{k}) = 0$$

$$\Rightarrow 3a_2 - a_3 = 0 \dots(ii)$$

$$\vec{d} \cdot \vec{c} = 1 \Rightarrow (a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}) (7\hat{i} - \hat{k}) = 1$$

$$\Rightarrow 7a_1 - a_3 = 1 \dots(iii)$$

Solving equation (i) and (ii) we get $3a_1 - a_3 = 0 \dots(iv)$

Again solving equation (iii) & (iv) we get $a_1 = \frac{1}{4}$

From equation (i), $a_1 - a_2 = 0$ or $a_1 = a_2 = \frac{1}{4}$

From equation (ii), $3a_2 - a_3 = 0 \Rightarrow 3 \cdot \frac{1}{4} = a_3 \Rightarrow a_3 = \frac{3}{4}$

$$\text{Hence, } \vec{d} = \frac{1}{4} \hat{i} + \frac{1}{4} \hat{j} + \frac{3}{4} \hat{k}$$

$$31. \text{ We have, } (\cos x)^y = (\cos y)^x$$

On taking log both sides, we get

$$\log(\cos x)^y = \log(\cos y)^x$$

$$\Rightarrow y \log(\cos x) = x \log(\cos y)$$

On differentiating both sides w.r.t x, we get

$$y \cdot \frac{d}{dx} \log(\cos x) + \log \cos x \cdot \frac{d}{dx}(y)$$

$$= x \frac{d}{dx} \log(\cos y) + \log(\cos y) \frac{d}{dx}(x) \text{ [by using product rule of derivative]}$$

$$\Rightarrow y \cdot \frac{1}{\cos x} \frac{d}{dx}(\cos x) + \log(\cos x) \frac{dy}{dx} = x \cdot \frac{1}{\cos y} \frac{d}{dx}(\cos y) + \log \cos y \cdot 1$$

$$\Rightarrow y \cdot \frac{1}{\cos x} (-\sin x) + \log(\cos x) \cdot \frac{dy}{dx} = x \cdot \frac{1}{\cos y} (-\sin y) \frac{dy}{dx} + \log \cos y \cdot 1$$

$$\Rightarrow -y \tan x + \log(\cos x) \frac{dy}{dx} = -x \tan y \frac{dy}{dx} + \log(\cos y)$$

$$\Rightarrow [x \tan y + \log(\cos x)] \frac{dy}{dx} = \log(\cos y) + y \tan x$$

$$\therefore \frac{dy}{dx} = \frac{\log(\cos y) + y \tan x}{x \tan y + \log(\cos x)}$$

Section D

32. According to Given question, Region is $\{(x, y) : x^2 + y^2 \leq 4, x + y \geq 2\}$.

The above region has a circle with equation $x^2 + y^2 = 4$ (i)

centre of the given circle is (0, 0)

Radius of given circle = 2

The above region consists of line whose equation is

$$x + y = 2 \text{(ii)}$$

Point of intersection of line and circle is

$$\Rightarrow x^2 + (2 - x)^2 = 4 \text{ [from Eq. (ii)]}$$

$$\Rightarrow x^2 + 4 + x^2 - 4x = 4$$

$$\Rightarrow 2x^2 - 4x = 0$$

$$\Rightarrow 2x(x - 2) = 0$$

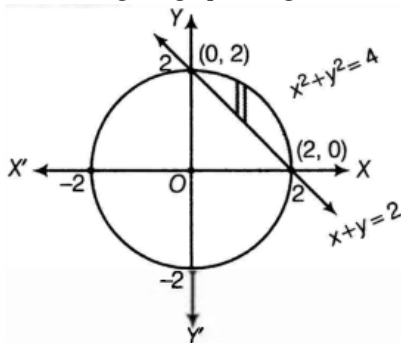
$$\Rightarrow x = 0 \text{ or } 2$$

When $x = 0$, then $y = 2 - 0 = 2$

When $x = 2$, then $y = 2 - 2 = 0$

So, points of intersection are (0, 2) and (2, 0).

On drawing the graph, we get the shaded region as shown below:



$$\text{Required area} = \int_0^2 [y_{(\text{circle})} - y_{(\text{line})}] dx$$

$$= \int_0^2 [\sqrt{4 - x^2} - (2 - x)] dx \text{ [From Eq(i) and (ii)]}$$

$$= \int_0^2 \sqrt{4 - x^2} dx - \int_0^2 (2 - x) dx$$

$$= \left[\frac{x}{2} \sqrt{4 - x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_0^2 - \left[2x - \frac{x^2}{2} \right]_0^2 \left[\because \int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x}{a} \right) \right]$$

$$= \left[0 + 2 \sin^{-1} \left(\frac{2}{2} \right) - 0 - 2 \sin^{-1} 0 \right] - \left(4 - \frac{4}{2} - 0 \right)$$

$$= (2 \sin^{-1} 1 - 0) - \left(4 - \frac{4}{2} \right)$$

$$= 2 \cdot \frac{\pi}{2} - 2$$

$$= (\pi - 2) \text{ sq units}$$

33. Given that $A = \{1, 2, 3, \dots, 9\}$ (a, b) R (c, d) $a + d = b + c$ for (a, b) $\in A \times A$ and (c, d) $\in A \times A$.

Let (a, b) R (a, b)

$$\Rightarrow a + b = b + a, \forall a, b \in A$$

Which is true for any $a, b \in A$

Hence, R is reflexive.

Let $(a, b) R (c, d)$

$$a+d = b+c$$

$$c + b = d + a \Rightarrow (c, d) R (a, b)$$

So, R is symmetric.

Let $(a, b) R (c, d)$ and $(c, d) R (e, f)$

$$a + d = b + c \text{ and } c + f = d + e$$

$$a + d = b + c \text{ and } d + e = c + f \Rightarrow (a + d) - (d + e) = (b + c) - (c + f)$$

$$(a - e) = b - f$$

$$a + f = b + e$$

$$(a, b) R (e, f)$$

So, R is transitive.

Hence R is an equivalence relation.

Let $(a, b) R (2, 5)$, then

$$a+5=b+2$$

$$a=b-3$$

If $b < 3$, then a does not belong to A .

Therefore, possible values of b are > 3 .

For $b=4, 5, 6, 7, 8, 9$

$$a=1, 2, 3, 4, 5, 6$$

Therefore, equivalence class of $(2, 5)$ is

$$\{(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)\}.$$

OR

$$A = R - \{3\} \text{ and } B = R - \{1\} \text{ and } f(x) = \left(\frac{x-2}{x-3}\right)$$

$$\text{Let } x_1, x_2 \in A, \text{ then } f(x_1) = \frac{x_1-2}{x_1-3} \text{ and } f(x_2) = \frac{x_2-2}{x_2-3}$$

$$\text{Now, for } f(x_1) = f(x_2)$$

$$\Rightarrow \frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$$

$$\Rightarrow (x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$$

$$\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 2x_1 - 3x_2 + 6$$

$$\Rightarrow -3x_1 - 2x_2 = -2x_1 - 3x_2$$

$$= x_1 = x_2$$

$\therefore f$ is one-one function.

$$\text{Now } y = \frac{x-2}{x-3}$$

$$\Rightarrow y(x-3) = x-2$$

$$\Rightarrow xy - 3y = x - 2$$

$$\Rightarrow x(y-1) = 3y-2$$

$$\Rightarrow x = \frac{3y-2}{y-1}$$

$$\therefore f\left(\frac{3y-2}{y-1}\right) = \frac{\frac{3y-2}{y-1}-2}{\frac{3y-2}{y-1}-3} = \frac{3y-2-2y+2}{2y-2-3y+3} = y$$

$$\Rightarrow f(x) = y$$

Therefore, f is an onto function.

$$34. \text{ Given } A^2 - 5A + 7I = 0$$

$$\text{L.H.S} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \text{R.H.S}$$

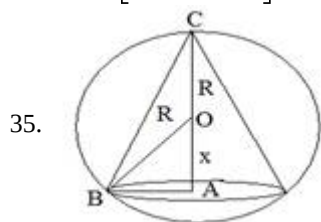
$$A^2 = 5A - 7I$$

$$A^3 = A^2 \cdot A$$

$$= 5A^2 - 7AI$$

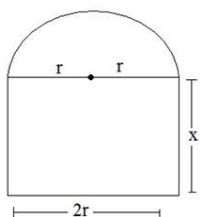
$$= 5A^2 - 7A \text{ (Since } AI = A)$$

$$\begin{aligned}
&= 5(5A - 7I) - 7A \\
&= 25A - 35I - 7A \\
A^3 &= 18A - 35I \\
A^4 &= A^3 \cdot A \\
&= (18A - 35I) \cdot A \\
&= 18A^2 - 35IA \\
&= 18(5A - 7I) - 35A \\
&= 90A - 126I - 35A \\
&= 55A - 126I \\
&= 55 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 126 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 165 & 55 \\ -55 & 110 \end{bmatrix} + \begin{bmatrix} -126 & 0 \\ 0 & -126 \end{bmatrix} \\
A^4 &= \begin{bmatrix} 39 & 55 \\ -55 & -16 \end{bmatrix}.
\end{aligned}$$



$$\begin{aligned}
v &= \frac{1}{3} \pi r^2 h \left[r^2 = \sqrt{R^2 - x^2} \right] \\
V &= \frac{1}{2} \pi \cdot (R^2 - x^2) \cdot (R + x) \\
\frac{dv}{dx} &= \frac{1}{3} \pi [(R^2 - x^2)(1) + (R + x)(-2x)] \\
&= \frac{1}{3} \pi [(R + x)(R - x) - 2x(R + x)] \\
&= \frac{1}{3} \pi (R + x)[R - x - 2x] \\
&= \frac{1}{3} \pi (R + x)(R - 3x) \dots (1) \\
\text{Put } \frac{dv}{dx} &= 0 \\
R &= -x \text{ (neglecting)} \\
R &= 3x \\
\frac{R}{3} &= x \\
\text{On again differentiating equation (1)} \\
\frac{d^2v}{dx^2} &= \frac{1}{3} \pi [(R + x)(-3) + (R - 3x)(1)] \\
&= \left[\frac{d^2v}{dx^2} \right]_{x=\frac{R}{3}} = \frac{1}{3} \pi \left[\left(R + \frac{R}{3} \right)(-3) + \left(R - 3 \cdot \frac{R}{3} \right) \right] \\
&= \frac{1}{3} \pi \left[\frac{4R}{3} \times -3 + 0 \right] \\
&= -\frac{4}{3} \pi R \\
\frac{d^2v}{dx^2} &< 0 \text{ Hence maximum} \\
\text{Now } v &= \frac{1}{3} \pi [(R^2 - x^2)(R + x)] \left[x = \frac{R}{3} \right] \\
v &= \frac{1}{3} \pi \left[\left(R^2 - \left(\frac{R}{3} \right)^2 \right) \left(R + \left(\frac{R}{3} \right) \right) \right] \\
&= \frac{1}{3} \pi \left[\frac{8R^2}{9} \times \frac{4R}{3} \right] \\
v &= \frac{8}{27} \left(\frac{4}{3} \right) \pi R^3 \\
v &= \frac{8}{27} \text{ Volume of sphere} \\
\text{Volume of cone} &= \frac{8}{27} \text{ of volume of sphere.}
\end{aligned}$$

OR



Let P be the perimeter of window

$$P = 2x + 2r + \frac{1}{2} \times 2\pi r$$

$$10 = 2x + 2r + \pi r \quad [P = 10]$$

$$x = \frac{10 - 2r - \pi r}{2}$$

Let A be area of window

$$A = 2rx + \frac{1}{2}\pi r^2$$

$$= 2r \left[\frac{10 - 2r - \pi r}{2} \right] + \frac{1}{2}\pi r^2$$

$$= 10r - 2r^2 - \pi r^2 + \frac{1}{2}\pi r^2$$

$$= 10r - 2r^2 - \frac{\pi r^2}{2}$$

$$\frac{dA}{dr} = 10 - 4r - \pi r$$

$$\frac{d^2A}{dr^2} = -(\pi + 4)$$

$$\frac{dA}{dr} = 0$$

$$r = \frac{10}{\pi + 4}$$

$$\frac{d^2A}{dr^2} < 0 \text{ maximum}$$

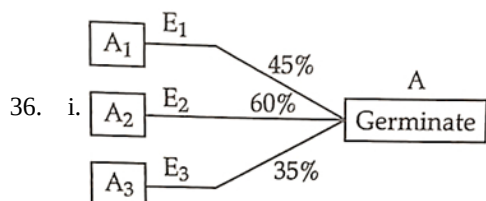
$$x = \frac{10 - 2r - \pi r}{2}$$

$$x = \frac{10}{\pi + 4}$$

$$\text{Length of rectangle} = 2r = \frac{20}{\pi + 4}$$

$$\text{width} = \frac{10}{\pi + 4}$$

Section E



$$\text{Here, } P(E_1) = \frac{4}{10}, P(E_2) = \frac{4}{10}, P(E_3) = \frac{2}{10}$$

$$P\left(\frac{A}{E_1}\right) = \frac{45}{100}, P\left(\frac{A}{E_2}\right) = \frac{60}{100}, P\left(\frac{A}{E_3}\right) = \frac{35}{100}$$

$$\therefore P(A) = P(E_1) \cdot P\left(\frac{A}{E_1}\right) + P(E_2) \cdot P\left(\frac{A}{E_2}\right) + P(E_3) \cdot P\left(\frac{A}{E_3}\right)$$

$$= \frac{4}{10} \times \frac{45}{100} + \frac{4}{10} \times \frac{60}{100} + \frac{2}{10} \times \frac{35}{100}$$

$$= \frac{180}{1000} + \frac{240}{1000} + \frac{70}{100}$$

$$= \frac{490}{1000} = 4.9$$

ii. Required probability = $P\left(\frac{E_2}{A}\right)$

$$= \frac{P(E_2) \cdot P\left(\frac{A}{E_2}\right)}{P(A)}$$

$$= \frac{\frac{4}{10} \times \frac{60}{100}}{\frac{490}{1000}}$$

$$= \frac{240}{490} = \frac{24}{49}$$

iii. Let,

E_1 = Event for getting an even number on die and

E_2 = Event that a spade card is selected

$$\therefore P(E_1) = \frac{3}{6}$$

$$= \frac{1}{2}$$

$$\text{and } P(E_2) = \frac{13}{52} = \frac{1}{4}$$

$$\begin{aligned} \text{Then, } P(E_1 \cap E_2) &= P(E_1) \cdot P(E_2) \\ &= \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8} \end{aligned}$$

OR

$$\begin{aligned} P(A) + P(B) - P(A \text{ and } B) &= P(A) \\ \Rightarrow P(A) + P(B) - P(A \cap B) &= P(A) \\ \Rightarrow P(B) - P(A \cap B) &= 0 \\ \Rightarrow P(A \cap B) &= P(B) \\ \therefore P(A|B) &= \frac{P(A \cap B)}{P(B)} \\ &= \frac{P(B)}{P(B)} \\ &= 1 \end{aligned}$$

37. i. Clearly, the coordinates of A are (8, 10, 0) and D are (0, 0, 30)

$\therefore$ Equation of AD is given by

$$\begin{aligned} \frac{x-0}{8-0} &= \frac{y-0}{10-0} = \frac{z-30}{-30} \\ \Rightarrow \frac{x}{4} &= \frac{y}{5} = \frac{30-z}{15} \end{aligned}$$

- ii. The coordinates of point C are (15, - 20, 0) and D are (0, 0, 30)

$\therefore$ Length of the cable DC

$$\begin{aligned} &= \sqrt{(0 - 15)^2 + (0 + 20)^2 + (30 - 0)^2} \\ &= \sqrt{225 + 400 + 900} = \sqrt{1525} = 5\sqrt{61} \text{ m} \end{aligned}$$

- iii. Since, the coordinates of point B are (-6, 4, 0) and D are (0, 0, 30), therefore vector DB is

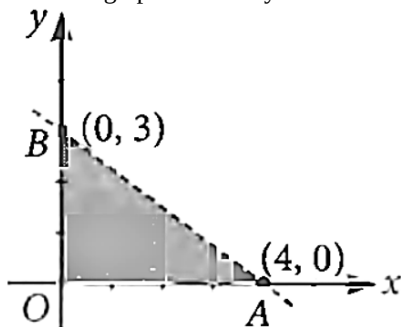
$$(-6 - 0)\hat{i} + (4 - 0)\hat{j} + (0 - 30)\hat{k}, \text{ i.e., } -6\hat{i} + 4\hat{j} - 30\hat{k}$$

OR

Required sum

$$\begin{aligned} &= (8\hat{i} + 10\hat{j} - 30\hat{k}) + (-6\hat{i} + 4\hat{j} - 30\hat{k}) + (15\hat{i} - 20\hat{j} - 30\hat{k}) \\ &= 17\hat{i} - 6\hat{j} - 90\hat{k} \end{aligned}$$

38. i. When we solve an L.P.P. graphically, the optimal (or optimum) value of the objective function is attained at corner points of the feasible region.
- ii. From the graph of $3x + 4y < 12$ it is clear that it contains the origin but not the points on the line $3x + 4y = 12$.



- iii. Maximum of objective function occurs at corner points.

Corner Points	Value of $Z = 2x + 5y$
(0, 0)	0
(7, 0)	14
(6, 3)	27
(4, 5)	33 ← Maximum
(0, 6)	30

OR

Value of $Z = px + qy$ at (15, 15) = $15p + 15q$ and that at (0, 20) = $20q$. According to given condition, we have $15p + 15q = 20q \Rightarrow 15p = 5q \Rightarrow q = 3p$